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An Empirical Study of Student Errors in Solving Absolute Value Inequalities and Function Limits

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Abstract
Solving absolute value inequalities and proving limits are foundational competencies in early calculus. However, many prospective mathematics teachers rely on procedural routines that fail when algebraic constraints and formal justification are required. This mixed-methods research combines a descriptive tally of written errors with follow-up clinical interviews to examine students' error patterns and the factors underlying them. Participants were 25 second-semester calculus students in a Mathematics Education program. A test on absolute value inequalities and limits (including an item requiring avoidance of division by zero and an ε–δ proof) was adapted from Purcell and Varberg and reviewed by three calculus lecturers for content relevance and clarity. Results indicate that routine limit evaluation by direct substitution was generally successful; however, performance dropped sharply for tasks demanding structural reasoning: no participant solved the limit item involving a removable discontinuity with a domain restriction, and only one participant produced an acceptable ε–δ argument. Across tasks, errors clustered into (i) conceptual errors (misinterpreting absolute value as "always positive" and conflating limit value with function value), (ii) systematic procedural errors (incorrect case-splitting, invalid algebraic transformations, and unjustified cancellation), and (iii) random/careless errors. Interview data suggest that incomplete concept images, fragile prerequisite algebra, and low confidence when facing proof-oriented prompts jointly contributed to these errors. The findings support instruction that explicitly foregrounds domain restrictions, connects multiple representations of limit, and scaffolds ε–δ reasoning through guided bounding and δ(ε) construction.

INTRODUCTION

Solving equations and inequalities has long been a core task in school mathematics, and these skills remain central in early university calculus. For prospective mathematics teachers, competence with absolute value inequalities and the limit of a function is not only procedural but also conceptual: students must interpret solution sets, attend to domain restrictions, and justify claims using definitions when required.

In practice, however, concepts of absolute value and limits are among the topics most frequently associated with misconceptions and fragile procedures. Research on absolute value shows that learners (including pre-service teachers) may treat |x| as a symbol that can be removed mechanically, confuse equations with inequalities, and apply case-splitting inconsistently, leading to incorrect solution sets (Altarawneh & Alkhateeb, 2024). Regarding limits, students' personal understandings often conflict with the formal epsilon-delta definition; many do not use the formal definition when solving limit problems, even after completing introductory calculus (Muzangwa & Chifamba, 2020; Orhun, 2022). Moreover, epistemological obstacles such as conflating a limit value with the function value and overgeneralizing substitution methods have been observed in first-year calculus, including in online learning environments (Sulastri et al., 2022).

These difficulties were also observed in the Mathematics Education Program at Universitas Tanjungpura. A preliminary survey of second-semester students indicated that many were unable to solve basic inequality tasks correctly and tended to avoid epsilon-delta reasoning by substituting values directly, even when the task structure demands algebraic reconstruction and formal justification.

Although student errors in inequalities and in limits have been widely discussed, the two domains are often examined separately. However, both require careful coordination of algebraic structure, domain restrictions, representation, and justification. Recent research on limit comprehension emphasizes that misconceptions and reasoning patterns vary with representation and the type of argument students produce, which makes detailed error analysis of written work, complemented by interview evidence, particularly informative for identifying teachable moments in calculus instruction (Slavickova & Vargova, 2023). Therefore, integrating error analysis across these foundational topics can yield actionable insights for calculus instruction in teacher education.

Accordingly, this research aims to (1) identify patterns of student errors in absolute value inequalities and function limits, and (2) explore the cognitive and affective factors behind these errors through carefully selected clinical interviews.

METHOD

This study investigated the types of errors students make when solving absolute value inequalities and limit problems, as well as the reasons behind these errors. It used a mixed-methods approach: quantitative analysis of written responses to identify common error patterns, followed by qualitative clinical interviews to understand why the errors happened. The participants were 25 second-semester calculus students from the Mathematics Program at Universitas Pamulang, with no students repeating the course.

Data were collected through a written test and follow-up clinical interviews. The test items were adapted from Purcell et al. (2021) and covered (i) absolute value inequalities and (ii) limits at a point, including items that require algebraic manipulation to avoid division by zero and items that necessitate an ε–δ proof. Before administration, three calculus lecturers reviewed the items for relevance, representativeness, and clarity, serving as expert judgment as evidence of content validity. Six students were then purposively selected for interviews to represent diverse written work profiles, including dominant conceptual errors, systematic procedural errors, random or careless errors, and variations in overall performance.

Analysis was conducted concurrently with data collection through transcription, data reduction, coding, classification, and synthesis. Students' written solutions were first grouped into answer models (MJ codes) and then aggregated into three error categories: conceptual, systematic, and random by aligning the observed patterns with established error classification ideas (e.g., Hadar et al., 1987; Clement, 1982) and with the notion of misconceptions and relational vs. instrumental understanding (Kastberg et al., 2020; Skemp, 1982). To improve trustworthiness, two researchers independently coded a subset of scripts and discussed discrepancies to reach consensus before finalizing the coding scheme.

RESULT AND DISCUSSION

The Results and Discussion section reports the dominant error patterns observed in students' written work on absolute value inequalities and function limits, and it triangulates these patterns with interview explanations. Written responses were first organized into answer models for each item and then coded into conceptual, systematic, and random/careless error categories.

To make the quantitative tendencies explicit, manifested in students' reasoning and algebraic manipulation.

Table 1. Summarizes the Assessed Tasks

Item	Target concept	Expected reasoning/strategy
Problem 1 (a–c)	Absolute value inequalities	Translate $\|A\| \lesseqqgtr k$ into compound inequalities/case analysis; apply domain restrictions.
Question 2 (a)	Limit evaluation	Algebraic manipulation (factoring/rationalization) to avoid division by zero; then evaluate.
Question 2 (b)	Limit evaluation with restriction	Identify undefined points, factor/simplify, and state domain restriction explicitly.
Problem 3 (a–b)	ε–δ definition of limit	Provide a bound to relate $\|x-a\| < \delta$ to $\|f(x)-L\| < \varepsilon$; justify $\delta(\varepsilon)$.

Table 2. Summarizes The Observed Distribution of Responses for The Limit and Proof-Related Items

Item	Correct	Incorrect / Incomplete	No answer	Note
Question 2(a) (limit value)	19	2	4	Four students left the item blank; two could not complete factoring; the remaining students obtained the expected value.
Question 2(b) (limit with restriction)	0	21	4	No student reached a correct final answer.
Problem 3(a) (ε–δ proof)	1	24	0	Only one student produced an acceptable ε–δ proof.
Problem 3(b) (ε–δ proof)	0	25	0	No student produced an acceptable ε–δ proof.

Table 3. Provides Representative (Anonymized) Excerpts of Written Work to Illustrate the Major Error Categories

Error Category	Typical Symptom	Student Answer Model (Code)	Interpretation / Theoretical Link
Conceptual error (Absolute Value)	Removes $\|\cdot\|$ without case-splitting; treats $\|x\|$ as mere brackets	MJ1A-3 (n=2): $\|x/3 - 2\| \leq 6$ $\Longrightarrow x/3 + 2 \leq 6$ $\Longrightarrow -2 \leq x/3 \leq 4$ $\Longrightarrow -6 \leq x \leq 12$	Student ignores the definition $\|a\| = a$ if $a \geq 0$, $-a$ if $a<0$. Misinterprets absolute value as a sign-removal operator. Aligns with Soedjadi's (1995) description of misconceptions caused by incomplete prior concept images.
Conceptual error (invalid operation)	Squares both sides of the inequality incorrectly, ignoring the sign	MJ1A-7 (n=1): $(x/3 - 2) \leq 6$ $\Longrightarrow (x/3 - 2)^2 \leq 6^2$ $\Longrightarrow 3x/3 - 6 \leq 108$ $\Longrightarrow x \leq 114$	Applies squaring as an arbitrary transformation; loses the direction of the inequality. Reflects instrumental use of rules without relational understanding (Skemp, 1982; Orhun, 2022).
Systematic procedural error (domain restriction)	Omits $x \neq 0$ constraint; ignores negative branch in solution	MJ1B-6 (n=5): $2 + 5/x > 1$ $\Longrightarrow (2x + 5)/x > 1$ [cross-mult without checking sign of x] $\Longrightarrow 2x + 5 > x$ $\Longrightarrow x > -5$ HP = $\{x \mid x > -5\}$	Most frequent systematic error (5 of 25 students). Cross-multiplication applied without case-checking the sign of x, producing an incomplete solution set missing $x < -5/3$ and the restriction $x \neq$ > 0. Consistent with Hadar et al. (1987) and Radmehr & Drake (2020), procedural routine errors.

Systematic procedural error (incorrect case-split)	Applies |a| > b ⟹ a > b OR a < b (forgetting sign flip)	MJ1B-7 (n=1): 2 + 5/x > 1 ⟹ 2 + 5/x > 1 OR 2 + 5/x < 1 [not < −1] ⟹ 5/x > −1 OR 5/x < −1 ⟹ 5 > −x OR 5 < −x ⟹ x > 5 OR x < −5	Student applies the wrong threshold for the negative case (uses −b = −1 but with the wrong inequality direction). Repeated consistently across similar problems; classified as systematic error (Clement, 1982).
Random/ careless error	Algebraic slip: adds numerators across unrelated terms	MJ1B-2 (n=2): 2 + 5/x > 1 ⟹ 7/x > 1 [treats 2 + 5/x as (2+5)/x] ⟹ 7 > 1	Isolated arithmetic error — combining whole numbers and fractions as if they share a denominator. Not a stable misconception; interview confirmed student knew the rule but 'was not careful'—Aligns with Clement's (1982) careless-error category.
Conceptual error (limit definition)	Uses direct substitution as ε–δ proof; confuses evaluation with justification	MJ3A (n=23): Prove lim(x→3) 3√x = √3: Students write: substitute x = 3: 3√3 = √3 ✓ [asserted as 'proven']	23 of 25 students substituted instead of constructing a δ(ε) argument. Reflects confusion between verifying a value and formally proving a limit. Students rely on surface procedures (Sulastri et al., 2022) and lack relational understanding of the ε–δ definition (Skemp, 1982).
Systematic error (ε–δ proof structure)	Writes definition correctly but cannot construct δ(ε); skips bounding step	MJ3B (n=1): Prove lim(x→3) x² − 2x = 3: Assume ε = |x − 2| − 3/x x − 2 − 3/x = ε x² − 6 = ε ∴ δ = x² − 6 = 9 − 6 = 3	Student sets δ as a specific numeric value rather than a function of ε; fails to perform the required bounding of |f(x) − L| in terms of |x − a|. This is the only student who attempted a structured proof, yet the bounding strategy is absent — consistent with lack of scaffolding for δ(ε) construction (Slavickova & Vargova, 2023).

Problem 1 (a): Determine the solution set (SS) of $|\frac{x}{3} - 2| \leq 6$

a. 1st Answer Model (MJ1A-1)

$$|\frac{x}{3} - 2| \leq 6$$
$$|-\frac{1}{3}| \leq 6$$
$$-\frac{1}{3} \leq 6$$

b. 2nd Answer Model (MJ1A-2)

$$|\frac{x}{3} - 2| \leq 6$$
$$\left|\frac{x}{3} - 2\right| - |6| \leq 0$$
$$\left|\frac{x}{3}\right| - |2| - |6| \leq 0$$
$$|\frac{x}{3}| \leq 18$$
$$|x| \leq |24|$$
$$x \leq 24$$

c. 3rd Answer Model (MJ1A-3)

$$\left|\frac{x}{3} - 2\right| \leq 6$$
$$\frac{x}{3} + 2 \leq 6$$
$$-2 \leq \frac{x}{3} \leq 4$$
$$-6 \leq x \leq 12$$
$$HP = \{x| -6 \leq x \leq 12\}$$

d. 4th Answer Model (MJ1A-4)

$$|\frac{x}{3} - 2| \leq 6$$
$$-4 \leq \frac{x}{3} \leq 8$$
$$-12 \leq x \leq 24$$
$$HP = \{x| -12 \leq x \leq 24\}$$

e. 5th Answer Model (MJ1A-5)

$$|\frac{x}{3} - 2| \leq 6$$
$$|\frac{x}{3}| \leq 6 + 2$$
$$\frac{x}{3} \leq 8$$
$$x \leq 8 - 3$$
$$x \leq 5$$

f. 6th Answer Model (MJ1A-6)

$|\frac{x}{3} - 2| \leq 6$
$\frac{1}{3}|x-2| \leq 6$
$|\frac{1}{3}||x-2| \leq 6$
$|\frac{x}{3} - 2| \leq \frac{6}{\varepsilon}$

g. 7th Answer Model (MJ1A-7)
$|\frac{x}{3} - 2| \leq 6$
$(\frac{x}{3} - 2) \leq 6^2$
$(\frac{x}{3} - 2) \leq 36$
$\frac{3x}{3} - 6 \leq 108$
$x \leq 114$
$HP = \{x \mid x \leq 114\}$

h. 8th Answer Model (MJ1A-8)
$|\frac{x}{3} - 2| \leq 6$
$-6 \leq \frac{x}{3} - 2 \leq 6$
$-6 + 2 \leq \frac{x}{3} \leq 6 + 2$
$-4 \leq \frac{x}{3} \leq 8$
$-\frac{4}{3} \leq x \leq \frac{8}{3}$

Problem 1 (b): Determine the solution set (SS) of $\left|2 + \frac{5}{x}\right| > 1$

a. 1st Answer Model (MJ1B-1)
$\left|2 + \frac{5}{x} - 1\right| > 0$
$\left|1 + \frac{5}{x}\right| > 0$
$\left(1 + \frac{5}{x}\right)^2 > 0$
$x^2 + 10x + 25 > 0$
$(x+5)(x+5) = 0$
$x = -5$

b. 2nd Answer Model (MJ1B-2)
$\left|2 + \frac{5}{x}\right| > 1$
$\left|\frac{7}{x}\right| > 1$
7>1

c. 3rd Answer Model (MJ1B-3)
$\left|2 + \frac{5}{x}\right| > 1$
$|2| + \left|\frac{5}{x}\right| > 1$
$2 + \frac{5}{x} > 1$
$\frac{5}{x} > -1$
$\frac{5}{x}\, x > (-1)x$
$5 > -x$
$x > -5$
$HP = (-5, \sim)$

d. 4th Answer Model (MJ1B-4)
$\left|2 + \frac{5}{x}\right| > 1$
$2 + \frac{5}{x} > 1 \quad \text{atau} \quad 2 + \frac{5}{x} < -1$
$\frac{5}{x} > -1 \quad \text{atau} \quad \frac{5}{x} < -3$
$5 > x \quad \text{atau} \quad -\frac{5}{3} < x$
$HP = \{x \mid -5 > x \text{ atau } -\frac{5}{3} < x\}$

e. 5th Answer Model (MJ1B-5)
$\left|2 + \frac{5}{x}\right| > 1$
$2 + \frac{5}{x} > 1 \quad \text{atau} \quad 2 + \frac{5}{x} < -1$
$\frac{2x+5}{x} > 1 \quad \text{atau} \quad \frac{2x+5}{x} < -1$
$2x + 5 > x \quad \text{atau} \quad 2x + 5 < -x$
$5 > -x \quad \text{atau} \quad 3x < -5$
$x > -5 \quad \text{atau} \quad x < -\frac{5}{3}$
$HP = (-5, -\frac{5}{3})$

f. 6th Answer Model (MJ1B-6)
$\left|2 + \frac{5}{x}\right| > 1$
$\left(2 + \frac{5}{x}\right) > 1^2$
$x\left(2 + \frac{5}{x}\right) > 1.x$
$2x + 5 > x$
$x > -5$
$HP = \{x|x > -5\}$

g. 7th Answer Model (MJ1B-7)
$\left|2 + \frac{5}{x}\right| > 1$
$2 + \frac{5}{x} > 1 \quad \text{atau} \quad 2 + \frac{5}{x} < 1$
$\frac{5}{x} > -1 \quad \text{atau} \quad \frac{5}{x} < -1$
$5 > -x \quad \text{atau} \quad 5 < -x$
$x > 5 \quad \text{atau} \quad x < -5$

Problem 1 (c): Determine the solution set (SS) of $|3x - 5| < 2|x + 4|$

a. 1st Answer Model (MJ1C-1)
$|3x - 5| < 2|x + 4|$

$|-2x| < 2|4x| = 2(4x)$
$|-2x| < 8x$
$2x < 8x$

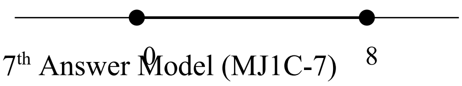
b. 2nd Answer Model (MJ1C-2)
$|3x-5| < 2|x+4|$
$3x-5 < 2x+4$
$3x < 2x+9$
$3x-2x < 9$
$x < 9$

c. 3rd Answer Model (MJ1C-3)
$-2(x+4) < 3x-5 < 2(x+4)$
$-2x-8 < 3x-5 < 2x+8$
$-2x-3 < 3x-5 < 2x+13$
$-1 < x < \frac{13}{3}$

d. 4th Answer Model (MJ1C-4)
$|3x-5| < |2||x+4|$
$|3x-5| < |2x+8|$
$|3x|-|5| < |2x|+|8|$
$|3x|-|2x| < |8|+|5|$
$|x| < |13|$
$x < 13$
$HP = (-\sim, 13)$

e. 5th Answer Model (MJ1C-5)
$|3x-5| < 2|x+4|$
$\left|\frac{3x-5}{x+4}\right| < 2$ atau $\left|\frac{3x-5}{x+4}\right| > -2$
$3x-5 < 2x+8$ atau $3x-5 < -2x-8$
$x < 13$ atau $x > -\frac{3}{5}$

f. 6th Answer Model (MJ1C-6)
$|3x-5-2| < |x+4|$
$|3x-7| < |x+4|$
$|3x-7-4| < |x|$
$3x-11 < x$
$3-11 < x-x$
$8 < 0$



g. 7th Answer Model (MJ1C-7)
$|3x-5| < |2||x+4|$
$|3x-5| < |2x+8|$
$3x-5 < 2x+8$
$3x-2x < 8+5$
$x < 13$
$HP = (13, \sim)$

From the students' responses for test question number 1, at least eight distinct error models were identified for part (a), and seven distinct answer models were found for each of parts (b) and (c). This count does not include instances where no answer was provided.

For problems that involve finding the limit of a function at a specific point, nearly all students perform well. The difficulty only arises when the solution requires algebraic manipulation to avoid division by zero. The question provided is as follows:

1. Question 2 (a): Find the value $\lim_{x\to -1} \frac{2x^2-x-3}{x+1}$
2. Question 2 (b); Find the value $\lim_{x\to 9} \frac{x-9}{\sqrt{x}-3}$
3. Problem 3 (a): Provide a proof using the ε-δ definition to show that $\lim_{x\to 3}\sqrt{3x} = 3$
4. Problem 3(b): Provide a proof using the ε-δ definition to show that $\lim_{x\to 3}(x^2-2x) = 3$

In answering both problems, almost all students approached them the same way as Problem 2. They substituted the limit value for the variable, just confirming the result. However, the goal was for them to demonstrate the validity of the limit value as the problem required. This shows that the students not only misunderstood the question's purpose but also could not prove it using the definition.

The different types of incorrect answers for each question were given specific codes. For instance, for Question 1, the error models are designated as MJ1A, MJ1B, and MJ1C for parts (a), (b), and (c), respectively; for Questions 2 and 3, the codes are MJ2A, MJ2B, MJ3A, and MJ3B, respectively. If the total number of students providing an answer is denoted by f, and the number of students whose answer falls under a specific error model is indicated by writing (f) after the error model code, then the distribution of error models for each question

For Question 2 (determining a function's limit), four students did not provide an answer, and two students failed to complete the solution due to improper factorization.

For Question 3 (proving a limit using the ε-δ definition), only one student provided the correct proof. Nevertheless, for the second part of the question, Question 2(b), no student was able to solve it. The failure in the limit proofs is attributed to the student's lack of understanding of the necessity of restricting a particular factor, indicating an overall lack of understanding of the limit proof method.

Based on the data obtained from the test on absolute value inequalities and function limits, which was further supplemented by clinical interviews with six purposively selected students, the following information was revealed:

Although these students had previously learned the concept of absolute value in high school, and it was revisited in their differential calculus course, they continued to make errors when solving inequalities involving absolute values. The mistakes mainly involved conceptual errors caused by a lack of understanding of what absolute value means. Some students removed the absolute value signs without proper case reasoning, and this error was seen repeatedly, indicating a systematic pattern. Representative excerpts are provided in Table 3.

From the range of responses labeled MJ1A-1 through MJ1A-8, MJ1B-1 through MJ1B-7, and MJ1C-1 through MJ1C-7, it is clear that the students did not master the concept of absolute value. A closer investigation revealed that the mistake was due to forgetting the concept. Additionally, the students failed to consider that the problem required certain conditions, such as the fact that division by zero is undefined. They did not consider the possibility that x could be any real number, including 0.

Selected interview excerpts are presented below as Figures 1–4 to illustrate the four dominant error categories. Each excerpt was translated and lightly edited for clarity.

Interview Excerpt 1 — Conceptual Error on Absolute Value (MJ1A-3)

Researcher:
"Mengapa Anda langsung menghapus tanda mutlak tanpa membagi menjadi dua kasus?"

Student (MJ1A-3):
"Saya pikir tanda mutlak itu sama seperti kurung biasa, Bu. Jadi saya langsung saja hilangkan dan lanjutkan hitung."

Researcher:
"Apakah Anda tahu definisi nilai mutlak?"

Student (MJ1A-3):
"Tahu Bu, tapi waktu ngerjakan soal saya tidak ingat harus dibagi dua kondisi. Saya rasa langsung saja bisa."

Figure 1. Interview Excerpt Showing Conceptual Error on Absolute Value Interpretation (MJ1A-3)

Interview Excerpt 2 — Systematic Error: Domain Restriction Omitted (MJ1B-6)

Researcher:
"Anda mengalikan silang dengan x — mengapa langsung mengalikan tanpa memeriksa tanda x?"

Student (MJ1B-6):
"Saya biasa langsung kalikan silang, Bu. Tidak sadar kalau x bisa negatif dan harus ganti arah pertidaksamaan."

Researcher:
"Jika x negatif, apa yang terjadi pada pertidaksamaan?"

Student (MJ1B-6):
"Oh... tandanya harus dibalik ya Bu. Berarti jawaban saya tidak lengkap. Saya lupa kasus $x < 0$ dan $x \neq 0$."

Figure 2. Interview Excerpt Showing Systematic Error Due To Omitted Domain Restriction (MJ1B-6)

Interview Excerpt 3 — Conceptual Error: Substitution as Proof (MJ3A, n=23)

Researcher:
"Apakah membuktikan limit berbeda dengan menghitung nilai limit?"

Student (MJ3A):
"Menurut saya sama, Bu. Kalau sudah disubstitusi dan hasilnya cocok, berarti sudah terbukti."

Researcher:
"Apa yang dimaksud dengan pembuktian formal menggunakan ε dan δ?"

Student (MJ3A):
"Saya belum pernah pakai cara itu, Bu. Di kelas kami hanya diajari substitusi langsung untuk menghitung limit."

Figure 3. Interview Excerpt Showing Conceptual Error: Direct Substitution Treated as Formal Proof (MJ3A)

Interview Excerpt 4 — Systematic Error: ε-δ Proof Incomplete (MJ3B)

Researcher:
"Anda sudah menulis definisi ε-δ dengan benar. Bagaimana Anda menentukan nilai δ?"

Student (MJ3B):
"Saya substitusi nilai x = 3 ke dalam ekspresi, Bu, lalu hasil hitungannya saya jadikan δ."

Researcher:
"Apakah δ harus bergantung pada ε?"

Student (MJ3B):
"Seharusnya ya Bu, tapi saya belum tahu caranya. Di langkah mana saya harus hubungkan δ dengan ε?"

Figure 4. Interview Excerpt Showing Incomplete ε–δ Proof Construction (MJ3B)

When interviewed, several students acknowledged that they often worked quickly and relied on memorized procedures, which increased the risk of overlooking key conditions (such as $x \neq 0$) and making local algebraic slips. Overall, the error patterns can be interpreted as a combination of misconceptions, repeated procedural routines that do not incorporate necessary constraints (Hadar et al., 1987; Radmehr & Drake, 2020), and careless inaccuracies (Clement, 1982; Peng et al., 2020). In terms of understanding, many responses suggest an instrumental use of rules rather than relational meaning-making (Skemp, 1982; Radmehr & Drake, 2020), especially when tasks required definition-based justification (ε–δ) rather than routine computation.

CONCLUSION

This research shows that errors in absolute value inequalities and function limits among second-semester Mathematics Education students cluster into three recurring forms: conceptual errors (misinterpreting the meaning of absolute value and inequality structure, and conflating limit values with function values), systematic procedural errors (invalid algebraic transformations, inconsistent case analysis, unjustified cancellation, and neglect of domain restrictions such as division by zero), and random/careless errors (inattention to signs, intervals, or algebraic details). Interview evidence suggests that incomplete concept images, fragile prerequisite algebra, and low confidence when tasks demand justification jointly contribute to these patterns.

Pedagogically, the findings point to three concrete actions in early calculus for prospective teachers: (1) incorporate short, structured error-analysis reflections using anonymized student solutions to make reasoning norms explicit; (2) foreground domain restrictions as a routine part of solution checking for inequalities and limits (especially when expressions can become undefined); and (3) scaffold epsilon-delta proofs through guided bounding, worked examples of delta(epsilon) selection, and gradual fading of support before independent proof construction. Future research can extend this work longitudinally to track how students' error profiles change across subsequent calculus courses and to evaluate the impact of targeted instructional interventions.

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